École Centrale de Nantes

EU-CORE Master Programme

European Master on Control of Renewable Energy Systems

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NOLCO Lab - Part I

Objective

The objective of this lab consists in designing some nonlinear controllers for wind turbines for which aerodynamic aspects are taken into account, as the electrical part that is a permanent magnet synchronous machine (PMSG). The work is focused on control in Region III, *i.e.* regulation of the power at its rated value.

1 Aerodynamic part control

A nonlinear model of the aerodynamic part reads as

$$\dot{\omega} = \frac{1}{J}\left[\tau_a - n_g\tau_g\right] + \delta(t) \tag{1}$$

in which ω is the rotor speed of the wind turbine, τ_a is the aerodynamic torque generated by the energy extracted from the wind, τ_g is the generator torque, and the term $\delta(t)$ is representing all the perturbations and uncertainties (including the friction force, neglected dynamics, ...). The parameter J is the inertia of the system whereas n_g is the gearbox ratio. The aerodynamic torque is defined as

$$\tau_a = \frac{P_a}{\omega} \tag{2}$$

with the aerodynamic power P_a reading as

$$P_a = \frac{1}{2}\zeta\pi R^2 C_p(\lambda,\beta)v^3 \tag{3}$$

In the previous equation, the parameter ζ is the air density, R the blade length, v the rotor-effective wind speed, and C_p the power coefficient that depends on the pitch angle β and the tip-speed ratio λ reading as

$$\lambda = \frac{\omega R}{v} \tag{4}$$

The power coefficient C_p can be expressed by several manners; among these latters, one has

$$C_p(\lambda,\beta) \quad \equiv \quad a(\cdot)\beta + b(\cdot) \tag{5}$$

with

$$\begin{aligned} a &= -c_0 c_2 e^{-c_4\lambda_1^{-1}} \\ b &= c_0\left(c_1\lambda_1^{-1} - c_3\right)e^{-c_4\lambda_1^{-1}} \\ \lambda_1^{-1} &= \frac{1}{\lambda + c_5\beta} - \frac{c_6}{\beta^3+1} \end{aligned} \tag{6}$$

Given the values of the parameters, and the definition of the both functions $a(\cdot)$ and $b(\cdot)$, it yields that the equation (1) can be rewritten as

$$\dot{x}_\omega \quad = \quad \mathcal{F}_\omega(\cdot) + \mathcal{G}_\omega(\cdot)\cdot u \tag{7}$$

with $x = \omega$, $u = [\tau_g \;\; \beta]^\top$ and

$$\mathcal{F}_\omega(\cdot) \quad = \quad \frac{\zeta\pi R^3 v^2}{2J\lambda}b(\cdot) + \delta(\cdot), \qquad \mathcal{G}_\omega(\cdot) \; = \; \left[-\frac{n_g}{J} \quad \frac{\zeta\pi R^3 v^2}{2J\lambda}a(\cdot)\right] \tag{8}$$

Remark 1 *The parameters of the previous system are detailed in Appendix A.* ■

Control Design Tasks.

1. Suppose first that $\delta(\cdot) = 0$. Design a control law based on input-output linearization approach, in order to force the rotor speed ω converging to ω^*, the so-called "rated rotor speed" that is defined as

$$\omega^* \quad = \quad \frac{P^*}{n_g \tau_g^*}$$

with P^* the rated power and τ_g^* the rated generator torque (see their values in Appendix).

2. Suppose now $\delta(t) \neq 0$ and is defined as

$$\delta(t) \quad = \quad 0.01 + 0.2\sin(0.5t) \tag{9}$$

. In this case, what is the behaviour of the closed-loop system ?

3. Now, considering the perturbed system, design a controller based on first order sliding mode control, including an equivalent control term. Tune the gain in order to converge towards the objective by taking into account the limit on blade pitch (magnitude, rate).

4. Finally, consider a dynamic gain for the sliding mode controller (see Slide #40 of SMC chapter). Check the interest and the consequences on the closed-loop system performances (for different values of gain rate ($\bar{K}$) or targeted accuracy (ε)). What happens if the parameter ε is chosen too small ?

5. In order to reduce the high-frequency oscillations (*chattering*), design now a super-twisting algorithm (slide #12 of SMC2 chapter).

How to present the simulations results ?

All the controllers must be evaluated on the provided Simulink model. Choose a reasonable time duration of the simulations, in order to have a good view of the transient. Use reasonable number of Figures.

NOLCO Lab - Part II

2 Electrical part control

In the previous section, only the aerodynamic part has been considered through the variable τ_g. Furthermore, it has been supposed that the generator torque τ_g is maintained at its rated value. In this second part, the objective is to introduce the model of the electrical part, and to propose a full scheme allowing the control of both electrical and aerodynamic part.

As previously claimed, the wind turbine is supposed to be equipped by a PMSG that is modelled by

$$\begin{aligned} \dot{i}_d &= -\frac{R_s}{L_d} i_d + \frac{pL_q}{L_d}\omega_g i_q + \frac{1}{L_d} V_d \\ \dot{i}_q &= -\frac{R_s}{L_q} i_q - \frac{pL_d}{L_q}\omega_g i_d - \frac{p\phi_f}{L_q}\omega_g + \frac{1}{L_q} V_q \end{aligned} \tag{10}$$

with $\omega_g = n_g \omega_r$. The link between the both part is made thanks to the rotation speeds.

Remark 2 *The parameters of the electrical machine are detailed in Appendix A.* ■

The control objectives of the electrical part are twofold

- force the direct current i_d to $i_d^* = 0$ in order to reduce the electromagnetic torque oscillations;
- force the quadrature current i_q to its reference i_q^* allowing to get the rated power P^*. The desired torque τ_g^* reads as

  $$\tau_g^* = \frac{P^*}{n_g \omega} \tag{11}$$

  Furthermore, considering the PMSG and given that $L_d = L_q$, the generator torque reads as

  $$\tau_g = \frac{3}{2} p \phi_f i_q \tag{12}$$

  Therefore, the generator torque can be adjusted by the quadratic current i_q, and indirectly controls the power output. Hence, if the current i_q tracks the following reference

  $$i_q^* = \frac{2P^*}{3 n_g \omega_r p \phi_f} \tag{13}$$

  so the power output is limited to its rated value.

Control Design Tasks.

1. Suppose first that $\delta(\cdot) = 0$. Design a control scheme based in input-output linearization approach, in order to reach the three objectives

   - $\omega \to \omega^*$;
   - $i_d \to 0$;
   - $i_q \to i_q^*$.

   Draw a figure of the full control scheme in order to show the integration of the control of both electrical and mechanical parts.

2. Suppose now that $\delta \neq 0$ and is similar as (9). Consider also parameters errors on R_s (-50%) and L_d, L_q (-20%). What is the behaviour of the system ?

3. Finally, design an adaptive supertwisting algorithm based on (see slide #17 of SMC2 chapter)

$$\begin{aligned} u &= -k_1|S|^{\frac{1}{2}}\mathrm{sign}(S) + v \\ \dot{v} &= -k_2\mathrm{sign}(S) \end{aligned} \tag{14}$$

   with k_1 and k_2 following the rules of Algorithm # 3 (see slide #19 of SMC2 chapter). Analyse the behaviour of the closed-loop system versus the choices made for the adaptation law parameters (ε, α, τ the sampling period).

A Wind Turbine Parameters

Physical, aerodynamic and electrical parameters

Symbol	Description	Value
R	Rotor radius (blade length)	63 m
$A = \pi R^2$	Swept area	1.2469×10^4 m^2
ζ	Air density	1.225 kg/m^3
n_g	Gearbox ratio	97
J	Equivalent inertia	3.8759228×10^7 kg m^2
R_s	Stator resistance	1.06 Ω
L_d, L_q	Direct/quadrature inductance	14.29 mH = 0.01429 H
ϕ_f	Flux linkage	8.6 Wb
p	No. of pole pairs	5

Table 1: Main physical and aerodynamic parameters of the wind turbine.

Rated operating point and control limits (Region III)

Symbol	Description	Value
P^*	Rated electrical power	5 MW
ω^*	Rated rotor speed	12.1 rpm (1.267 rad/s)
ω_g^*	Rated generator speed	1173.7 rpm
τ_g^*	Rated generator torque	4.309355×10^4 N m
$\dot{\tau}_{g,\max}$	Maximum generator torque rate	1.5×10^4 N m/s
$\beta_{\min}$	Minimum pitch angle	0°
$\beta_{\max}$	Maximum pitch angle	90°
$\dot{\beta}_{\max}$	Maximum pitch rate	8°/s

Table 2: Rated operating point and actuator constraints used for control design.

Power coefficient model

The coefficients of the C_p model are summarized in Table 3.

c_0	c_1	c_2	c_3	c_4	c_5	c_6
0.5	73.5	0.4	5	13.125	0.08	0.035

Table 3: C_p-law coefficients used in the aerodynamic model.

B MATLAB code

Listing 1: MATLAB code of the aerodynamic plant model

```
function [omega_dot, Power] = Plant(omega, v, beta, delta)

% Parameters
ng        = 97;
taugStar  = 43093;      % N.m (generator rated torque)
R         = 63;         % m
rho       = 1.22;       % kg/m^3
J         = 38759228;   % kg.m^2

% Tip-speed ratio
lambda = (omega * R) / v;

% Coefficients
c0 = 0.5;  c1 = 73.5; c2 = 0.4; c3 = 5;
c4 = 13.125; c5 = 0.08; c6 = 0.035;

u = beta;

% Aerodynamic Cp approximation (depends on lambda and beta)
lambda1_inv = 1/(lambda + c5*u) - c6/(u^3 + 1);
a = -(c0*c2) * exp(-c4*lambda1_inv);
b =  (c0*(c1*lambda1_inv - c3)) * exp(-c4*lambda1_inv);
Cp = a*u + b;      % Cp in affine-form w.r.t. beta
Cp = min(max(Cp, 0), 0.5);

%%%%%%%%%%%%%%%%%%%%%%%%%
Power = Tg * ng * omega;   % Aerodynamic power
%%%%%%%%%%%%%%%%%%%%%%%%%

% ---- F + G*u form ----
F = (1/J) * ( ((0.5 * rho * pi * R^3 * v^2)/lambda)*b - ng*taugStar );
G = (1/J) * ( ((0.5 * rho * pi * R^3 * v^2)/lambda)*a );

omega_dot = F + G*u + delta;

end
```