SIMUS

Student Name: ______________________ **Date:** __________________

1. Consider a linear time-invariant (LTI) system described by

$$\dot{x} = Ax + Bu, \qquad y = Cx.$$

   (a) A full-state feedback controller of the form $u = -Kx$ is applied to the system. Derive the resulting **closed-loop state equation** and clearly show the expression for the **closed-loop system**. Hence, identify the **closed-loop poles** of the controlled system.

   (b) A Luenberger observer is designed for the same system, given by

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}), \qquad \hat{y} = C\hat{x}.$$

   Derive the **estimation error dynamics**, obtain the corresponding **closed-loop error system**, and identify the **observer poles**.

2. Consider the nonlinear system

$$\dot{x}_1 = \cos(x_3)\, u_1, \tag{1}$$

$$\dot{x}_2 = \sin(x_3)\, u_1, \tag{2}$$

$$\dot{x}_3 = u_2. \tag{3}$$

This system describes the dynamic behavior of a mobile robot in the plane, with x_1 and x_2 being the Cartesian coordinates. The objective is to control the two variables x_1 and x_2.

To achieve this, a nonlinear dynamic control is implemented and defined by

$$\begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \cos(x_3) & -\sin(x_3)\, u_1 \\ \sin(x_3) & \cos(x_3)\, u_1 \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{4}$$

(a) Compute the two inputs v_1 and v_2 given that their purpose is to stabilize the following two subsystems:

$$\ddot{x}_1 = v_1 \;=\; -k_{11}\, \dot{x}_1 \;-\; k_{10}\, x_1, \tag{5}$$

$$\ddot{x}_2 = v_2 \;=\; -k_{11}\, \dot{x}_2 \;-\; k_{20}\, x_2. \tag{6}$$

(b) Set up a simulator that stabilizes the system at the origin. Plot the trajectory of the system in the plane (x_1, x_2), as well as the control inputs u_1 and u_2.

3. Find and consider a linear time-invariant (LTI) system, such as an **inverted pendulum**, **DC motor**, or any other controllable system, represented by

$$\dot{x} = Ax + Bu, \qquad y = Cx.$$

(a) Design a **state-feedback controller** of the form $u = -Kx$ by **pole placement** (using Ackermann's formula or any suitable pole placement technique) to obtain the desired closed-loop pole locations as you want.

(b) Design and tune PID controller for the same system using **PidTune**.

(c) Design a **Linear Quadratic Regulator (LQR)** controller by selecting appropriate weighting matrices Q and R. Compute the optimal gain matrix K_{LQR}.

(d) Apply each controller (**PID**, **State Feedback**, and **LQR**) to the same input, for example a **unit step input** or a reference trajectory. Compare the following performance indices: Rise time, Overshoot, Settling time, Steady-state error.

(e) Discuss the main differences among these controllers.